Piecewise affine modeling and explicit model predictive control for non-inverting buck-boost DC-DC converter[1]

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Abstract. An innovative EMPC (Explicit Model Predictive Control) method based on piecewise affine modeling is proposed for the non-inverting Buck-Boost DC-DC converter to overcome the control difficulties encountered in practice. Based on a piecewise affine model, the proposed EMPC method divides the state space into critical regions by multi-parametric programming and solves the traditional MPC optimal problem explicitly and fully off-line. As a result, the time-taking on-line iteration algorithm of MPC is replaced by a simple table lookup algorithm which greatly reduces the on-line computation time. Reliable control performance is achieved with respect to system constraints and hybrid dynamics. Compared with conventional controllers, the proposed EMPC method achieves better performance by accurate predictive model and coordinate control which is verified by simulation.

Key words. Piecewise affine modeling, explicit model predictive control, Buck-Boost DC-DC converter.

1. Introduction

DC-DC switched power converters are used to transform an unregulated DC voltage input to a regulated output DC voltage which have been widely studied and become a mature and well-established technology [1]. The Buck-Boost converter is

a combination of basic Buck and Boost DC converter topology which is successfully implemented in a wide range of electric power supply systems. This paper mainly focuses on the modeling and control problem of Buck-Boost converter. Most of conventional converters are regulated by simple analog controllers including a traditional PI (Proportional Integral) controller and a PWM (Pulse Width Modulation) unit [2]. Due to the inevitable dynamic of the analog PI regulator and PWM, conventional controllers are not suitable for high efficiencies demanding applications. In recent years, digital control strategies (such as H∞ methods, fuzzy control, nonlinear techniques, sliding mode control and so on), have emerged as an increasingly viable option for DC-DC converters with the fast development of the digital computational power [3].

The difficulty of the controller design for Buck-Boost converters stems from their hybrid and nonlinear nature, non-minimum phase behavior, the constraints of system states and so on. Among these modern control strategies, one research direction with significant potential is coordinated control using a MPC algorithm which have already been proposed in the DC-DC converter control [4, 5]. In the literature, MPC is regarded as an efficient control strategy based on the completely multivariable system framework [6]. In spite of its ability of optimal and constraint handling, the application of MPC strategy needs expensive on-line computation power and MPC is labeled as a technology for slow processes. Recently, EMPC is proposed to handle this problem [7, 8]. EMPC moves all the computations necessary for the implementation of MPC off-line using multi-parameter programming, while preserving all its other characteristics. Therefore, EMPC reduces on-line computation time and renders MPC suitable for fast systems such as switched power converters. For DC-DC power converters, EMPC has been studied in parallelized DC-DC converter [9] and electrical drives [10]. As for more complicated Buck-Boost converter, the application of EMPC strategies is still under investigation and more suitable model is needed.

A piecewise affine discrete-time modeling method, which is accurate and more adequate for EMPC paradigm to be applied, is proposed for non-inverting Buck-Boost DC-DC converter in this paper. Based on a piecewise affine discrete-time model, EMPC strategy is developed to reduce the on-line computation time and regulate the output voltage under system mismatches and disturbances. EMPC based on the piecewise affine model achieves more accurate and reliable performance compared with averaged model. As a result, the closed loop performance is improved and the complexity of controller is greatly reduced. Besides these benefits, the proposed EMPC respects all the constraints of the Buck-Boost system which is difficult to handle in the conventional control strategies.

2. Methodology

2.1. Circuit topology

An overview of the Buck-Boost DC-DC converter is given in this section to illustrate the background of the control problem. Fig. 1 depicts the circuit topology

and physical setup of the Buck-Boost converter where S_i are switches. while V_{s} and V_{rmo} represent the input and output voltages, respectively. Quantities D_i are the diodes and R_{o} is the load resistance. Symbols X_{l} and X_{c} are the inductance of the inductor and capacitance of the capacitor, respectively. Symbols R_{l} and R_{c} stand for the parasitic equivalent series resistances of the inductor and capacitor, respectively. Quantity T_{s} denotes the switching period and f_{s} is the corresponding switching frequency. Finally, d represents the duty cycle and $V_{\mathrm{o,ref}}$ denotes the reference value of the output voltage.
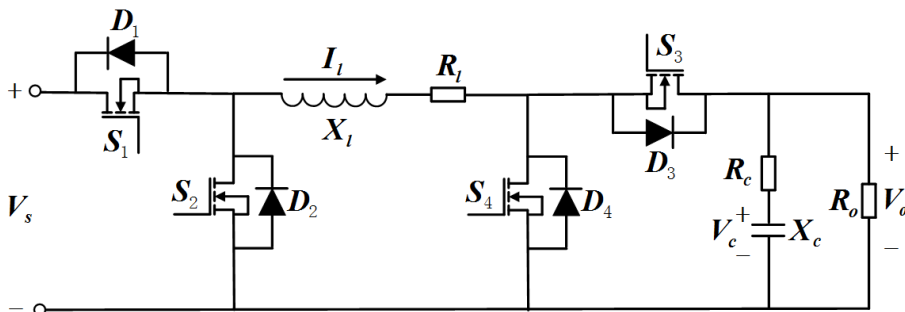

Fig. 1. Physical setup of Buck-Boost DC-DC converter

In Buck-Boost converters, couple transistors S_1 and S_2, (S_3 and S_4) work in a complementary manner, that is, when switch S_1 (S_3) is on, switch S_2 (S_4) must be off, and vice-versa. All of the switches are operated in a cyclic manner and duty cycle d is usually defined as $d = t_{\mathrm{on}}/T_{\mathrm{s}}$ where T_{s} is the switching period and t_{on} is the working time interval of S_1. The control objective is adjusting the duty cycle $d(k)$ to reach the output voltage reference. Buck-Boost DC-DC converter has two distinct dynamical modes: $[kT_{\mathrm{s}}, \ kT_{\mathrm{s}} + d(k)T_{\mathrm{s}}]$ (Mode 1) and $[kT_{\mathrm{s}} + d(k)T_{\mathrm{s}}, \ (k+1)T_{\mathrm{s}}]$ (Mode 2).

2.2. Mathematical model

To obtain a suitable numerical model for controller design, the parameters are normalized according to the base quantities: $R_{\mathrm{b}} = R_0$, $L_{\mathrm{b}} = R_{\mathrm{b}}/(2\pi f_{\mathrm{s}})$, $C_{\mathrm{b}} = 1/(2\pi f_{\mathrm{s}}R_{\mathrm{b}})$, $I_{\mathrm{b}}(k) = V_{\mathrm{s}}(k)/R_{\mathrm{b}}$, which simplifies the mathematical description of the system [4]. Mathematical model of the non-inverting Buck-Boost DC-DC converter can be derived by choosing $x(t) \in [i_1(t), v_{\mathrm{o}}(t)]^{\mathrm{T}}$ as the state vector. By applying Kirchhoff's Voltage and Current Laws, we have the continuous time state space model for each mode.

Mode 1:

$$\dot{x}(t) = F_1 x(t) + g_1 , \qquad (1)$$

$$v_{\mathrm{o}} = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) . \qquad (2)$$

Mode 2:

$$\dot{x}(t) = F_2 x(t) + g_2 \,, \tag{3}$$

$$v_{\mathrm{o}} = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t) \,. \tag{4}$$

where

$$F_1 = 2\pi f_{\mathrm{s}} \begin{bmatrix} -\frac{r_1}{x_1} & 0 \\ 0 & -\frac{1}{x_{\mathrm{c}}(r_{\mathrm{o}}+r_{\mathrm{c}})} \end{bmatrix} \,, \quad g_1 = \begin{bmatrix} \frac{1}{x_1} \\ 0 \end{bmatrix} \,,$$

$$F_2 = 2\pi f_{\mathrm{s}} \begin{bmatrix} -\frac{1}{x_1} r_1 & -\frac{1}{x_1} \\ -\frac{r_1 r_{\mathrm{o}} r_{\mathrm{c}}}{x_1(r_{\mathrm{o}}+r_{\mathrm{c}})} + \frac{1}{x_{\mathrm{c}}} \left(\frac{r_{\mathrm{o}}^2}{(r_{\mathrm{o}}+r_{\mathrm{c}})^2} + \frac{r_{\mathrm{c}}}{r_{\mathrm{o}}+r_{\mathrm{c}}} \right) & -\frac{r_{\mathrm{o}} r_{rmc}}{x_1(r_{\mathrm{o}}+r_{\mathrm{c}})} - \frac{1}{x_{\mathrm{c}} r_{\mathrm{o}}} \end{bmatrix} \,,$$

$$g_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \,.$$

2.3. Piecewise affine modeling

Let us start from the normalized continuous state space model (1)–(4) and assume that the converter is either in mode 1 or mode 2 during the sampling period k, that is, either $d_k = d(k) = 1$ or $d_k = d(k) = 0$. Taking the switching period as the sampling period, the exact discrete-time state space update model is given as

$$x_{k+1} = F_{\mathrm{d1}} x_k + g_{\mathrm{d1}} \,, \ \text{for } d_k = 1 \,, \tag{5}$$

$$x_{k+1} = F_{\mathrm{d2}} x_k + g_{\mathrm{d2}} \,, \ \text{for } d_k = 0 \,. \tag{6}$$

Here $F_{\mathrm{d}i} = \mathrm{e}^{F_i T_{\mathrm{s}}}$, $i = 1, 2$ and $g_{\mathrm{d}i} = \int_0^{T_{\mathrm{s}}} \mathrm{e}^{F_i} \,\mathrm{d}t \cdot g_i$, $i = 1, 2$. For the general case $0 \le d_k \le 1$, we employ the weighted discrete-time modeling method, that is, mode 1 and mode 2 are weighted with respect to their effective duration. And the resulting weighted discrete-time model is as follows:

$$x_{k+1} = (F_{\mathrm{d1}} x_k + g_{\mathrm{d1}}) \, d_k + (F_{\mathrm{d2}} x_k + g_{\mathrm{d2}}) \, (1 - d_k) =$$

$$= A_{\mathrm{av}} x_k + B_{\mathrm{av}} x_k d_k + C_{\mathrm{av}} d_k + D_{\mathrm{av}} \,. \tag{7}$$

However, due to existence of the multiple item $x_k d_k$, the state space model (7) is nonlinear. For high degree of accuracy, a piecewise affine approximation of the system dynamic is proposed, resulting in a piecewise affine system of the form:

$$x_{k+1} = A_i x_k + B_i d_k + f_i, \ \ d_i \le d_k \le d_{i+1}, \ i = 1, \cdots, I \,, \tag{8}$$

where d_i and d_{i+1} represent the lower and upper bound values of the duty cycle interval for which the ith approximation is valid, with $d_1 = 0$ and $d_I = 1$. Approximation is applied by choosing appropriate d_i for selected duty cycle intervals.

For each duty cycle interval, d_k in the nonlinear term $x_k d_k$ can be replaced by the constant median value of the interval, that is,

$$x_{k+1} = \left(A_{\mathrm{av}} + B_{\mathrm{av}} \frac{(d_i + d_{i+1})}{2} \right) x_k + C_{\mathrm{av}} d_k + D_{\mathrm{av}},$$

$$d_i \leq d_k \leq d_{i+1}, \ i = 1, \cdots, I. \tag{9}$$

This achieves the piecewise affine system update model

$$A_i = A_{\mathrm{av}} + B_{\mathrm{av}} \frac{(d_i + d_{i+1})}{2}, \ B_i = C_{\mathrm{av}}, \ f_i = D_{\mathrm{av}},$$

$$d_i \leq d_k \leq d_{i+1}, ß = 1, \cdots, I, \tag{10}$$

where B_i and f_i can be determined by the steady-state operating point continuity constraints for the piecewise affine model consistently with the averaged model. To be specific, for the selected constant duty cycle d_i, the steady-state operating points calculated by the averaged model is as follows:

$$\bar{x}_{\mathrm{av},i} = (I_2 - A_{\mathrm{av}} - B_{\mathrm{av}} d_i)^{-1} (C_{\mathrm{av}} d_i + D_{\mathrm{av}}). \tag{11}$$

For the piecewise model, we have $\bar{x}_{\mathrm{pwa},i,i} = (I_2 - A_i)^{-1} (B_i d_i + f_i)$ and $\bar{x}_{\mathrm{pwa},i,i+1} = (I_2 - A_i)^{-1} (B_i d_{i+1} + f_i)$. The steady-state operating point continuity constraints are imposed by $\bar{x}_{\mathrm{pwa},i,i} = \bar{x}_{\mathrm{av},i}$ and $\bar{x}_{\mathrm{pwa},i,i+1} = \bar{x}_{\mathrm{av},i+1}$ which yields four equations for each piecewise approximation. Quantities B_i and f_i can be fully determined as

$$\left[\begin{array}{c} B_i \\ f_i \end{array} \right] = M_i^{-1} \left[\begin{array}{c} \bar{x}_{\mathrm{av},i} \\ \bar{x}_{\mathrm{av},i+1} \end{array} \right], M_i = \left[\begin{array}{c} (I_2 - A_i)^{-1} \left[\begin{array}{cccc} d_i & 0 & 1 & 0 \\ 0 & d_i & 0 & 1 \end{array} \right] \\ \\ (I_2 - A_i)^{-1} \left[\begin{array}{cccc} d_{i+1} & 0 & 1 & 0 \\ 0 & d_{i+1} & 0 & 1 \end{array} \right] \end{array} \right]. \tag{12}$$

As a result, a piecewise affine model is completely defined and is continuous across its boundaries. The approximation accuracy is determined by the duty cycle intervals.

2.4. EMPC based on multi-parametric programming: a brief review

A brief review of EMPC based on multi-parametric programming is given in this section [8]. Consider a discrete-time MIMO LTI (Linear Time-Invariant) system of the regular form:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k), \\ y(k) = Cx(k), \end{cases} \tag{13}$$

while subjecting to the constraints $x_{\min} \leq x(k) \leq x_{\max}$, $y_{\min} \leq y(k) \leq y_{\max}$, $u_{\min} \leq u(k) \leq u_{\max}$. In the literature, for system (13), MPC solves the following

optimization problem:

$$\min_{U \triangleq \{u_k, \cdots, u_{k+N_u-1}\}} J$$

where

$$J = \left\{ J\left(U, x(k)\right) = x'_{k+N_p} P x_{k+N_p} + \sum_{j=0}^{N_p-1} \left[x'_{k+j} Q x_{k+j} + u'_{k+j} R u_{k+j} \right] \right\}. \quad (14)$$

The idea of MPC is the construction of an optimal control input sequence $U^* = \left\{ u_k^*, u_{k+1}^*, \cdots, u_{k+N_u-1}^* \right\}$, which minimizes the cost function J in (14) with respect to the state, output and input constraints (13). And MPC employs the receding horizon control principle, only the first step of the control input U^* (i.e., $u^*(k)$) is taken into the system at the time instant k. As for $k+1$, the whole procedure will be repeated once again. In this paper, we adopt a recently proposed EMPC strategy based on multi-parametric programming which is able to move all the computations of MPC off-line [11]. To be specific, EMPC utilizes multi-parametric programming to systematically subdivide the space X of parameters x_k into critical regions (CRs). For each CR, the optimal solution is an affine function of x_k. Once the critical region CR_0 has been defined, the rest of the space $CR^{rest} \triangleq X \backslash CR_0$ can be explored and new critical regions will be generated by an iterative algorithm which partition CR^{rest} recursively [7, 8]. As a result, state space X are divided into critical regions, and in each region, the optimal solution is an affine function of x_k which is calculated off-line. EMPC is extremely suitable for the piecewise affine discrete-time model of the Buck-Boost converter.

3. Result analysis and discussion

3.1. EMPC controller design

Non-inverting Buck-Boost DC-DC converter is taken as an example to the implementation of EMPC strategies. And the parameters are given as follows: $x_l = 2.5133$, $x_c = 50.2655$, $r_c = 0.005$, $r_l = 0.025$, $r_o = 1$, $i_{l,\max} = 10$, $v_s = 1$, $v_{o,ref} = 0.75$, $v_{o,\max} = 2$. Piecewise affine discrete-time model can easily be derived from the parameters by choosing appropriate duty cycle intervals. The control objectives are to minimize the output voltage error $v_{o,err} = v_o - v_{o,ref}$ with respect to the constraints. Additionally, we introduce the difference of two consecutive duty cycles $\Delta d(k) = d(k) - d(k-1)$. Define the penalty matrix $Q = diag(q_1, q_2)$ and the vector $\varepsilon(k) = [v_{o,err}(k), \Delta d(k)]$. The performance index function is given as

$$J = \sum_{l=0}^{N-1} \varepsilon^T(k) Q \varepsilon(k). \quad (15)$$

Based on the piecewise affine discrete-time linear model and the performance index function (15), piecewise affine EMPC controllers can be designed according to

the standard design procedure.

3.2. Simulation result analysis and discussion

Dynamic simulations using Matlab were carried out to evaluate the performance of the proposed EMPC strategy based on piecewise affine model. For the sake of comparison, we also report simulation results of the conventional PI algorithm which employs output feedback and EMPC based on averaged model. As far as the characteristics of the Buck-Boost converter system are concerned, we choose prediction horizon $N = 5$ for EMPC. Weight matrix is selected as $Q = diag(1,5)$. And the constraints are chosen as $0 \leq d \leq 1$, $|\Delta d| \leq 0.2$. The duty cycle interval is selected as: $[0, 0.3]$, $[0.3, 0.7]$ and $[0.7, 1]$.

Simulation results of the Buck-Boost converter are given in Figs. 2–6, where the results of EMPC based on piecewise affine model (represent by PEMPC) are drawn in solid lines, EMPC based on averaged model (represent by EMPC) are drawn in dashed dot lines, PID controllers in dashed lines. The polyhedral partition of PEMPC are shown in Fig. 2. For each critical region, the optimal control law is an affine function of the states and the previous control input. Once the critical region of the state is determined, the optimal feedback control law is directly calculated. As a result, the online calculation time will be greatly reduced compared with traditional MPC approach.
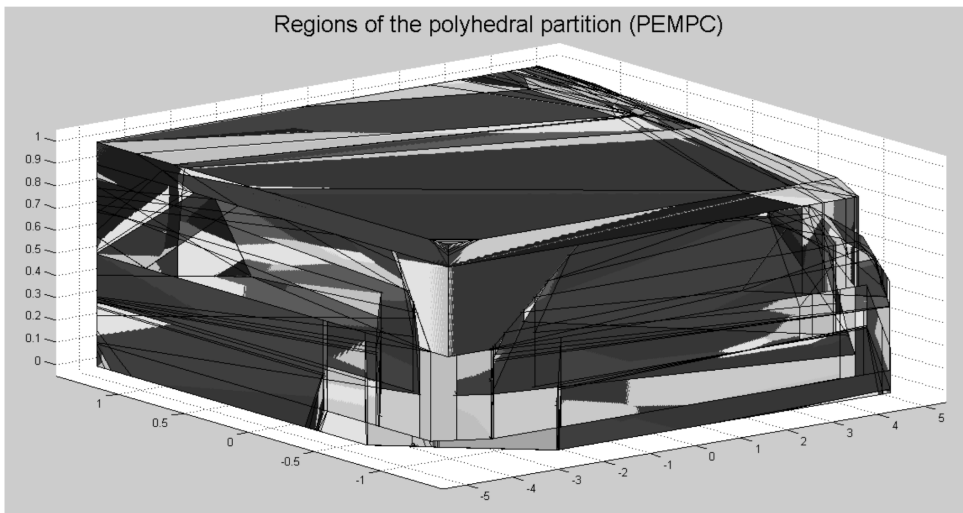

Fig. 2. Regions of the polyhedral partition calculated based on piecewise affine model

Figs. 3–4 give the closed-loop responses during startup with output voltage reference $v_{oref} = 0.4$ p.u. under PI, EMPC and PEMPC controllers. As we can see from the trajectories, EMPC has a steady state error caused by inaccurate averaged model which is overcomed by PEMPC. PEMPC controller derives the output voltage to the reference quickly and with small overshot whereas the PI controller reacts
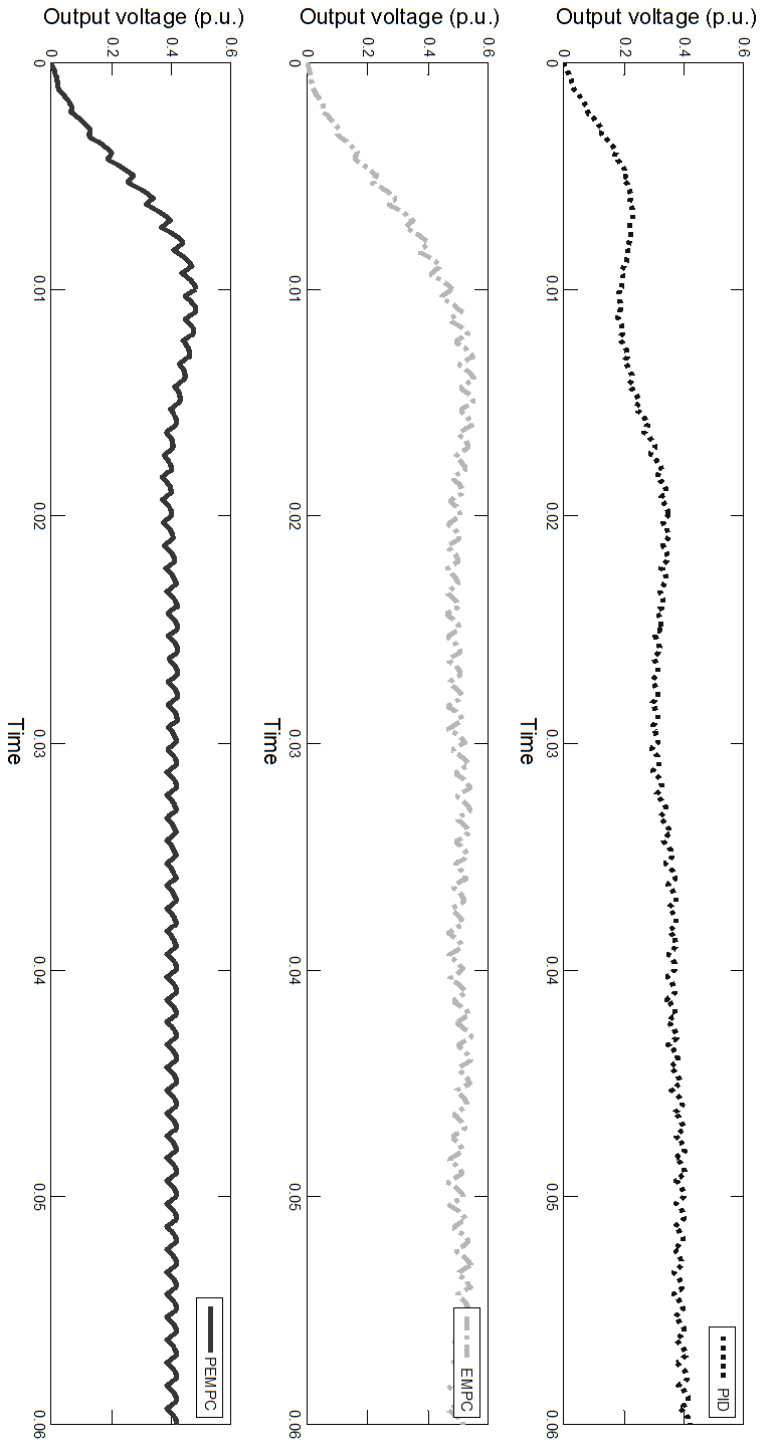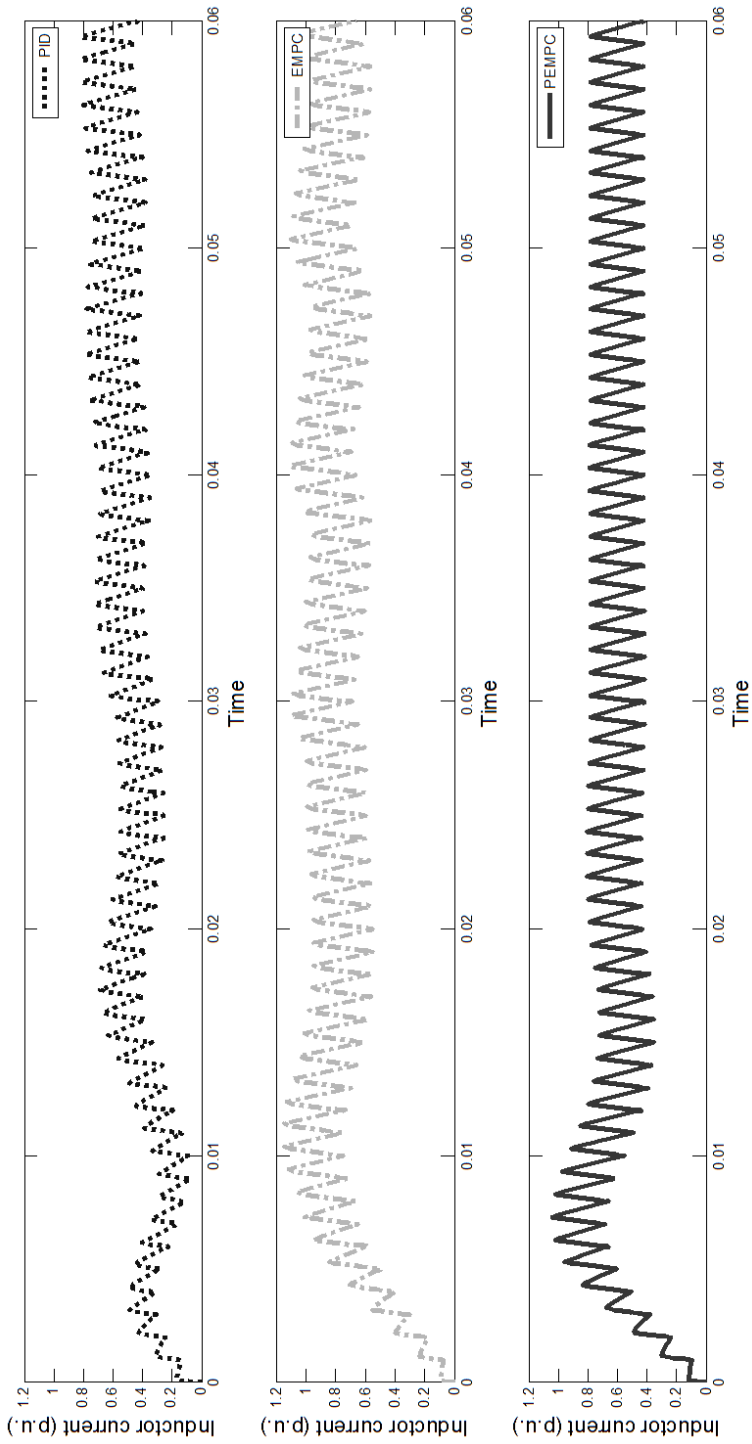
Fig. 3. Output voltage responses during startup
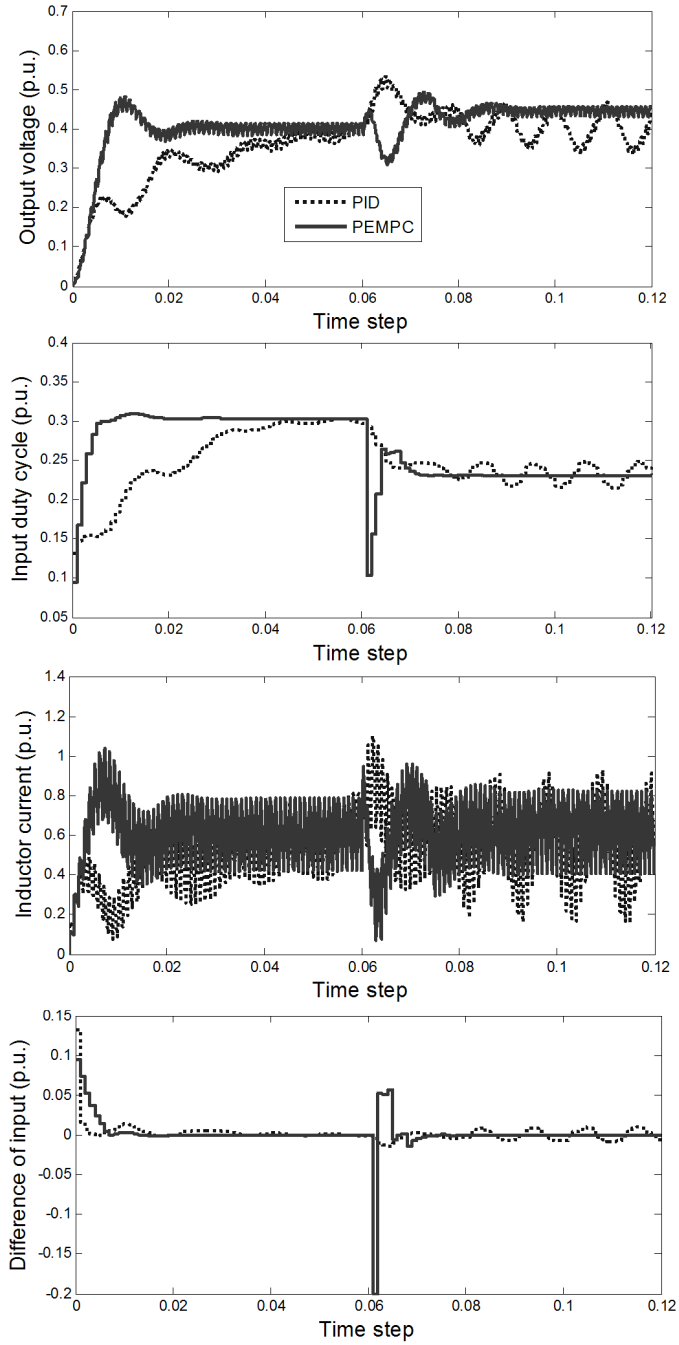
Fig. 4. Inductor current responses during startup

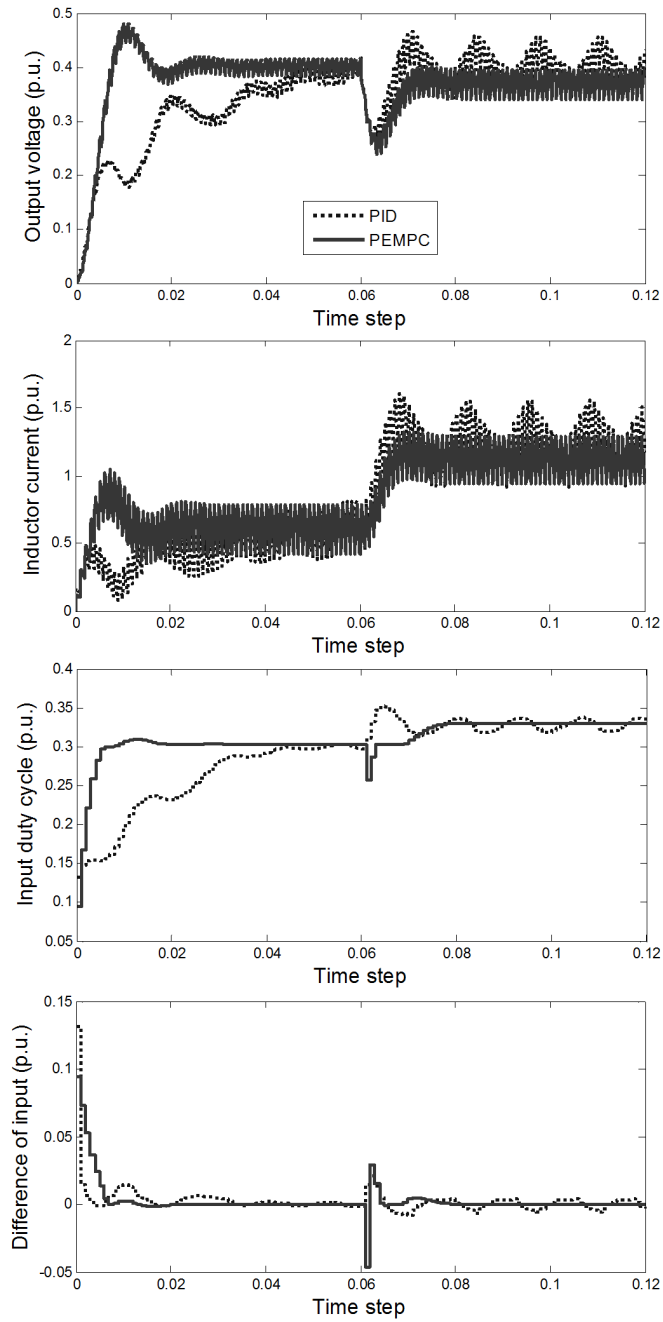Fig. 5. Closed-loop responses to the step-up change in the input voltage active for
$t \geq 0.06\,\mathrm{s}$

Fig. 6. Closed-loop responses to the step-down change in the load resistor active
for $t \geq 0.06\,\mathrm{s}$

slowly and results in big overshot. Closed-loop responses to several typical kinds of disturbances, which are frequently encountered in practice, are also analyzed. Since the input voltage source of Buck-Boost converter is unregulated, the variations in the input voltage is frequently encountered. Therefore, a step-up change in the input voltage from 1 p.u. to 1.4 p.u. after start up is simulated and the closed-loop responses are given in Fig. 5. Both PEMPC and PI controllers can derive the output voltage to the reference, however PEMPC provides less deviations in output voltage and inductor current. The output load may vary dramatically, so the variation in the load resistor is also investigated. Fig. 6 gives the closed-loop responses to the step-down change in the load resistor from 1 p.u. to 0.5 p.u. after start up. Trajectories of the system output and states show the effectiveness of PEMPC controller compared with traditional PI controller.

As we can see from the simulation results, EMPC based on piecewise affine model improves the closed-loop performance systematically and the controller is easy to tune by adjusting the weight matrices.

4. Conclusion

An innovative non-inverting Buck-Boost DC-DC converter control based on EMPC was investigated in this paper to overcome the control difficulties encountered in practice. A piecewise affine model is derived to model the hybrid and nonlinear system dynamic precisely. The traditional MPC optimal controller is transformed into a table lookup algorithm and the online computation is greatly reduced. As a result, Buck-Boost converter system could be precisely modeled and coordinately controlled by proposed EMPC. EMPC strategy investigated in this paper solves the control problem of high switching frequency converters systematically. It could be extended to converters with higher switching frequency and more complicated topology to get better control performances compared with existing ones.

However, in order to apply this scheme to industry practice, a major and important work to be done is realization of the algorithm in the embedded system which has limited computation power and hardware resources.

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